16.9 Videos Guide

16.9a

Theorem (statement and proof):

- The Divergence Theorem (the 3-D analog of Green's Theorem):
  $\iint_S \mathbf{F} \cdot d\mathbf{S} = \iiint_E \operatorname{div} \mathbf{F} \, dV$, where S is the boundary surface of E, a solid region whose surfaces are continuous, with outward orientation

Exercises:

16.9b

- Verify that the Divergence Theorem is true for the vector field $\mathbf{F}$ on the region E.
  $\mathbf{F}(x, y, z) = \langle x^2, -y, z \rangle$,
  E is the solid cylinder $y^2 + z^2 \leq 9$, $0 \leq x \leq 2$

16.9c

- Use the Divergence Theorem to calculate the surface integral $\iint_S \mathbf{F} \cdot d\mathbf{S}$; that is, calculate the flux of $\mathbf{F}$ across S.
  - $\mathbf{F}(x, y, z) = (x^3 + y^3)\,\mathbf{i} + (y^3 + z^3)\,\mathbf{j} + (z^3 + x^3)\,\mathbf{k}$,
    S is the sphere of radius 2 with center $(0, 0, 0)$

16.9d

  - $\mathbf{F}(x, y, z) = (xy + 2xz)\mathbf{i} + (x^2 + y^2)\,\mathbf{j} + (xy - z^2)\,\mathbf{k}$,
    S is the surface of the solid bounded by the cylinder $x^2 + y^2 = 4$ and the planes $z = y - 2$ and $z = 0$